A Simple Framework of the Firm and Information Channels

The framework behind "The Retreat of Local Banking: Spillover Effects on SME Credit"

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This note presents a simple framework of how a bank's loss of local presence affects the lending of a firm's other banks. It shows why a closure can lead a firm's lenders to cut together, through two channels: the borrower loses the closing bank's support, which makes it riskier for every lender (the borrower-risk channel), and the co-lenders lose the information that reached them from the closing bank (the information channel). It considers an *information shock* to one lender: for reasons unrelated to the firm, the lender monitors it less closely. The lender then learns less about the borrower, but the shock itself carries no news about the firm. The closure of a bank's last branch in a municipality is our example of an information shock; the departure of the loan officer who handled a relationship, or a reorganization that moves lending decisions away from the branch, are others. The framework spells out the distinct implications of each channel for the co-lenders' credit and for their own assessments of the firm, guiding our empirical analysis. It also explains why comparing the closing bank with the firm's other lenders understates the firm's loss. Section 1 sets out the environment, and Section 2 characterizes the equilibrium and states the results. Section 3 gives a numerical illustration and discusses the framework's scope, and Section 4 contains the proofs.

1 Setup

Two incumbent banks lend to a firm: bank A, which will be hit by the shock (the *shocked bank*), and a co-lender, bank B; j' denotes the bank other than j. Everyone is risk neutral, and there is no discounting within the lending cycle. The framework has four elements.

1. *Information shock.* Before credit is renewed, bank A's monitoring of the firm falls from m_A^0 to m_A^1. The shock is public and carries no news about the firm, whose viability for the coming cycle is drawn only at renewal.

2. *Credit lines.* At renewal, the banks simultaneously commit lines x_A and x_B in $[0, \bar{x}]$, where $\bar{x}$ is an exposure limit, for total credit $x = x_A + x_B$. The firm values committed credit at $rx - \psi x^2/2$ and takes the fee as given, so the market-clearing fee per euro of committed credit is $r - \psi x$, where $\psi > 0$ captures downward-sloping credit demand as banks compete to lend more; the fee is paid at renewal. Lending x_j euros costs bank j a linear funding cost $f_j x_j$ and a quadratic balance-sheet cost $\frac{\omega}{2} x_j^2$.

3. *Viability.* The firm is viable with probability v and nonviable with probability $1 - v$; no one observes the draw, the firm included. Time within the cycle is continuous, from renewal at $t = 0$ to a settlement date t_s, when the firm's payments fall due. Settlement arrives at rate d, independently of the firm's viability and of the banks' signals, and lines are drawn only to make these payments. At settlement, a nonviable firm always defaults, since it owes more than any credit could cover, whereas a viable firm defaults only if its liquidity need exceeds its credit lines. The need is zero with probability $1 - \ell_0$ and otherwise uniform on $[0, \ell_0/\varphi]$, so a viable firm defaults with probability $\ell(x) = \ell_0 - \varphi x$ for $x \leq \ell_0/\varphi$: credit from either bank supports the borrower. A bank loses the line it has open when the firm defaults.[1]

4. *Monitoring.* Before settlement, bank j's monitoring produces signals of a nonviable firm's distress, which arrive as a Poisson process with rate m_j, independently across the two banks; m_j reflects the bank's local presence, and a viable firm generates none. Signals are distress events in the bank's relationship with the firm, and they keep arriving after the bank freezes its line. Each signal reaches the other bank through the credit register with probability $\kappa \in (0, 1)$, independently of the others, so bank j learns of distress at rate $\Lambda_j = m_j + \kappa m_{j'}$. The share κ is effective: it covers register entries and visible changes in the other bank's exposure, and it summarizes reporting delays and the coarseness of the register; banking secrecy rules out passing soft information directly. A bank may freeze its line only after a distress signal, as covenants allow. A bank that learns of distress freezes its line and avoids any loss; if settlement comes first, the nonviable firm draws the line and the bank loses it.

[1] A loss given default $g < 1$ on every default scales the liquidity term, so that $\eta = vg\varphi - \psi$, and the information terms. A freeze that saves only part of the line adds a constant to each margin and scales the information terms. Neither changes the ratio ρ, and every result holds with the rescaled parameters.

Combining these four elements, bank j's expected profit at renewal is

$$\Pi_j = \underbrace{(r - \psi x - f_j)\, x_j}_{\text{net fee income}} - \underbrace{\frac{\omega}{2} x_j^2}_{\substack{\text{balance-sheet} \\ \text{cost}}} - \underbrace{v\, \ell(x)\, x_j}_{\substack{\text{loss if the firm is viable} \\ \text{but lacks liquidity}}} - \underbrace{(1-v)\, \frac{d}{d+\Lambda_j}\, x_j}_{\substack{\text{loss if the firm is nonviable} \\ \text{and the bank learns too late}}} . \qquad (1)$$

Maximizing Π_j over x_j gives bank j's interior best response, $x_j = a_j + \sigma x_{j'}$, where

$$a_j = \frac{r - f_j - v\ell_0 - (1-v)\, d/(d+\Lambda_j)}{\omega - 2\eta}, \qquad \sigma = \frac{\eta}{\omega - 2\eta}, \qquad \eta = v\varphi - \psi. \qquad (2)$$

We write M_j for the numerator of a_j, bank j's margin on the first euro of credit, and $G = \omega - 2\eta$, so that $a_j = M_j/G$ and $\sigma = \eta/G$. The other bank enters this best response in two ways. Its credit enters through the slope σ: each euro of it lowers the fee by ψ and the viable firm's default probability by φ, so it changes bank j's return on each euro of its own line by η. Its monitoring enters the stand-alone line a_j, the line bank j would extend if the other bank lent nothing, through Λ_j.

Three assumptions complete the framework: the first two drive the results, and the third collects regularity conditions.

Assumption 1 (Strategic complements) *$v\varphi > \psi$: the liquidity support that one bank's credit gives the other bank's claim outweighs the competition for the firm's business, so $\sigma > 0$.*

Assumption 2 (Imperfect monitoring) *$m_A^0 \leq d$: before the shock, the shocked bank's own monitoring detects distress before settlement in at most half of nonviable firms.*

Assumption 3 (Regularity) *(a) Each bank would lend on its own, before and after the shock: $M_j > 0$. (b) Credit never fully insures the firm, and the firm values every feasible line: $2\bar{x} \leq \ell_0/\varphi$ and $r \geq 2\psi\bar{x}$; equilibrium lines are below $\bar{x}$. (c) The firm is a good risk: $1 - v + v\ell_0 < 1/2$. (d) Balance-sheet costs bound the banks' mutual support: $\omega > 3\eta$, so that $\sigma < 1$.*

Assumption 1 ensures that the two credit lines act as strategic complements. This means that when one bank is affected and decides to cut its credit line, the co-lender also reduces its credit line instead of taking over the business. Assumption 2 ensures that the shocked bank loses more information than any co-lender, which received only a share κ of its signals. Without it, the shocked bank's information loss need not exceed the co-lender's: a bank that already caught most nonviable firms could lose little from monitoring less. In Assumption 3, parts (a), (b) and (d) keep both banks lending, keep the equilibrium interior and make it unique (Lemma 2); part (c) keeps the probability of default below one half on every borrower

without a distress signal, which the proof of Proposition 5(c) uses. Benchmarks without the register, $\kappa = 0$, are taken in economies that also satisfy Assumption 3.

2 Equilibrium and Results

The banks make two kinds of decisions: at renewal, they choose their lines simultaneously; within the cycle, each decides only whether to freeze its line. Lemma 1 describes the freeze decision and the beliefs behind it, and Lemma 2 the renewal game.

Lemma 1 (Decisions and beliefs within the cycle) *(a) Whatever the other bank does, it is optimal for a bank with a positive line to freeze it at its first distress signal, its own or one from the other bank; without a signal the contract does not allow a freeze. (b) Settlement comes before bank j learns of a nonviable firm's distress with probability $d/(d+\Lambda_j)$, and bank j freezes its line and classifies the firm in default before settlement with probability $P_j = \Lambda_j/(d+\Lambda_j)$. (c) At a date $t < t_s$, conditional on having received no signal about the firm, bank j puts probability $n_j(t)/(n_j(t)+v)$ on nonviability, where $n_j(t) = (1-v)e^{-\Lambda_j t}$, and its probability that the firm defaults at settlement is*

$$PD_j(t) = \frac{n_j(t) + v\,\ell(x)}{n_j(t) + v}. \tag{3}$$

After a signal, its probability of default is one: it classifies the firm in default.

Lemma 2 (The renewal game) *Under Assumptions 1 and 3, the renewal game has a unique Nash equilibrium. Bank j's best response is $a_j + \sigma x_{j'}$ clipped to $[0, \bar{x}]$, with $\sigma \in (0,1)$, and the equilibrium lines are $x_j = (a_j + \sigma a_{j'})/(1-\sigma^2) \in (0, \bar{x})$.*

Every change Δx_j below is the change in an equilibrium line, so it includes each bank's response to the other's.

The information shock reaches the banks' lines through their stand-alone lines. Superscripts 0 and 1 denote values before and after the shock. Comparisons across co-lenders, as in Propositions 2 and 3, and across firms, as in Proposition 4, compare otherwise identical two-bank economies.

Lemma 3 (What the information shock does to the stand-alone lines) *(a) An information shock of size $z = m_A^0 - m_A^1 > 0$ lowers both stand-alone lines, $\Delta a_j = -(1-v)D_j/G$, where*

$$D_A = \frac{d\,z}{(d+\Lambda_A^0)(d+\Lambda_A^1)}, \qquad D_B = \frac{d\,\kappa z}{(d+\Lambda_B^0)(d+\Lambda_B^1)} \tag{4}$$

are the increases in the probability that settlement comes before the bank learns of a nonviable firm's distress. The ratio of the falls in the two stand-alone lines,

$$\rho = \frac{\Delta a_B}{\Delta a_A} = \frac{D_B}{D_A} = \kappa \, \frac{(d + \Lambda_A^0)(d + \Lambda_A^1)}{(d + \Lambda_B^0)(d + \Lambda_B^1)}, \tag{5}$$

measures the co-lender's information loss relative to the shocked bank's. (b) (i) For a small shock between banks that monitor equally, $m_B = m_A^0$, $\rho \to \kappa$. (ii) ρ strictly decreases with the co-lender's own monitoring m_B. (iii) Under Assumption 2, $\rho < 1$ for every co-lender.

Propositions 1 to 3 trace how the equilibrium lines respond.

Proposition 1 (Both banks cut: the firm and information channels) *After an information shock of any size, both banks reduce their credit lines, in euros and in percent of their pre-shock lines. The co-lender's credit adjustment decomposes as*

$$\Delta x_B = \frac{\rho + \sigma}{1 - \sigma^2} \, \Delta a_A = \underbrace{\sigma \, \Delta x_A}_{\textit{borrower-risk channel}} + \underbrace{\Delta a_B}_{\textit{information channel}} < 0, \tag{6}$$

where both terms are negative:

(a) The borrower-risk channel, $\sigma \, \Delta x_A$, *captures the co-lender's reaction to the shocked bank's credit cut, which reduces the firm's liquidity buffer and heightens its risk;*

(b) The information channel, Δa_B, *captures the loss of information signals from the shocked bank that reached the co-lender through the register.*

Either channel suffices to induce a cut by the co-lender: without the register ($\kappa = 0$), only the borrower-risk channel operates; without credit-line complementarities ($\sigma = 0$), only the information channel operates. Analogously, the shocked bank's credit cut reflects both channels: $\Delta x_A = \sigma \, \Delta x_B + \Delta a_A$.

Proposition 1 is the framework's central result. In our application, the shocked bank is the closing bank. The borrower-risk channel measures the co-lender's response to the closing bank's total cut, which includes the closing bank's own response to the co-lender's information loss. Our empirical analysis estimates the co-lender's total cut and splits it between the two channels.

Proposition 2 (The shocked bank cuts more) *After an information shock, the shocked bank cuts more than every co-lender in euros,* $\Delta x_A - \Delta x_B = (1 - \rho) \, \Delta a_A / (1 + \sigma) < 0$. *Because the co-lender cuts too, this gap understates the shocked bank's own cut, and the firm's credit falls by more than the gap,* $\Delta x = (\Delta x_A - \Delta x_B) + 2\Delta x_B$.

Both banks respond in the same way to the fall in the firm's total credit, so the gap in euros keeps only the difference between their information losses (see the proof in Section 4). In percent of pre-shock lines, write $\Delta^{\mathrm{CB}} = \Delta x_A / x_A$ for the shocked bank, the closing bank in our application, $\Delta^{\mathrm{CL}} = \Delta x_B / x_B$ for the co-lender, and $s = x_A / x$ for the shocked bank's share of the firm's credit. The firm's percent change in credit is then

$$\frac{\Delta x}{x} = s\,\Delta^{\mathrm{CB}} + (1-s)\,\Delta^{\mathrm{CL}} = s\,\theta + \Delta^{\mathrm{CL}}, \qquad \theta = \Delta^{\mathrm{CB}} - \Delta^{\mathrm{CL}}. \tag{7}$$

Remark 1 (Proposition 2 in percent) *After an information shock, the shocked bank cuts more in percent, $\Delta^{\mathrm{CB}} < \Delta^{\mathrm{CL}}$, if and only if the shock lowers its stand-alone line by a larger percentage than the co-lender's, $\Delta a_A / a_A < \Delta a_B / a_B$, that is, $\rho < a_B / a_A$. Under this condition the within-firm gap θ lies between Δ^{CB} and zero. Since the co-lender cuts, $\Delta x / x = s\,\theta + \Delta^{\mathrm{CL}} < s\,\theta$. In every case $\Delta x / x = (\Delta M_A + \Delta M_B)/(M_A + M_B)$: for a given ℓ_0, it does not depend on ψ, φ or ω, which set σ.*

The condition reduces to $\rho < 1$ for banks with equal stand-alone lines. It holds for every co-lender that monitors at least as much as the shocked bank did before the shock and has no higher funding cost, and it holds in the illustration of Section 3. Because the co-lender cuts too, neither the within-firm gap θ nor its weighted version $s\,\theta$ measures the firm's loss (Section 3 gives an example).

Proposition 3 (Nonlocal co-lenders are hit harder) *After an information shock, compare co-lenders that differ only in their own monitoring m_B. Every co-lender cuts; the less it monitors, the more it cuts, in euros and in percent, and the larger the share of its cut that comes through the information channel. Without the register, $\kappa = 0$, every co-lender would cut by the same amount in euros.*

The borrower-risk channel reaches every co-lender, but a co-lender that monitors the firm itself relies less on the shocked bank's signals, so it loses less information when they fall. Our empirical analysis uses this contrast to separate the two channels, comparing co-lenders with and without a branch in the municipality in euros, since in percent the contrast would also reflect differences in pre-shock lines. The separation adds two assumptions, that local co-lenders lose no information and that the borrower-risk channel is the same at both. Figure 1 shows the co-lender's response and its two channels by its own monitoring.

Two further results describe which firms lose more co-lender credit, and what the shock does to the firm's default risk and to the banks' assessments of it.

Proposition 4 (Firms that depend more on the shocked bank) *After an information shock:*

(a) Credit shares. *Across firms with the same total credit that differ only in how the funding costs are split between the banks, f_A lower and f_B higher by the same amount, the co-lender's response in euros is the same, through each channel, and its cut in percent is larger the larger the shocked bank's share s.*

(b) Prior monitoring. *Across firms that differ only in how closely the shocked bank monitored them before the shock, m_A^0, with the same m_A^1, firms monitored more closely borrow more from both banks; after the shock both banks cut more, in euros and in percent, both parts of the co-lender's cut grow in euros, and the firm's default probability rises more. The shocked bank's share rises with m_A^0 if and only if $\kappa\,[(d+\Lambda_A^0)/(d+\Lambda_B^0)]^2 < a_B/a_A$, which holds near symmetric monitoring and funding costs.*

The fall in total credit also weakens the borrower, and the loss of signals changes what each bank knows about it. Call a co-lender *equally informed* if it monitored as much as the shocked bank did before the shock, $m_B = m_A^0$.

Proposition 5 (Borrower weakening and banks' assessments) *After an information shock:*

(a) Default risk. *Total credit falls, $\Delta x = (1+\rho)\,\Delta a_A/(1-\sigma) < 0$, so the firm's default probability, and both banks' probabilities of default at renewal, $1 - v + v\,\ell(x)$, rise by $v\varphi|\Delta x|$.*

(b) Classification and assessments. *Both banks become less likely to classify a nonviable firm in default before settlement, and the shocked bank more so, $\Delta P_A - \Delta P_B = -(1-\rho)D_A < 0$. Against an equally informed co-lender, at every date after renewal: on nonviable firms, the shocked bank's average probability of default is lower; conditional on having received no signal, its probability of default is higher, so that on a borrower neither bank has classified, viable firms included, its probability is the higher one; and over all borrowers before settlement, the two banks' average probabilities of default are equal.*

(c) Forecast accuracy. *At every date before settlement, each bank's expected squared forecast error rises, over all borrowers and over the borrowers it has not classified. Against an equally informed co-lender, the shocked bank's rises more at every date after renewal, over all borrowers and over each bank's own unclassified borrowers; at renewal the two are equal.*

(d) Reported PDs. *Suppose each bank revises its reported probability of default on a borrower only at renewal and when it classifies the borrower in default. Against an equally informed co-lender, the shocked bank's average reported probability of default then falls relative to the co-lender's at every date after renewal.*

Part (d) concerns the closing bank's own reported PD. With Bayesian probabilities of default the two banks' averages are equal, by part (b), because the shocked bank's fewer default classifications are offset by higher probabilities on the borrowers it has not classified. Under the reporting rule of part (d), which the framework takes as given, the offset is gone, and the co-lender's own reported PD rises only if borrower weakening outweighs its slower classification.

3 Illustration and Scope

Figure 1 and Table 1 illustrate the results for an information shock that lowers the shocked bank's monitoring from $m_A^0 = 1$ to $m_A^1 = 0.25$. The parameters, listed in the notes to the table, are illustrative, not calibrated. Each change is the total change in an equilibrium line. For a co-lender without monitoring of its own, the borrower-risk channel accounts for 0.70 percentage points of its 1.95 percent cut and the information channel for 1.25; for a co-lender that monitors as much as the shocked bank did before the shock, the cut is 0.80 percent, of which 0.45 points come through the borrower-risk channel. In euros, the borrower-risk channel is about one and a half times as large at the first co-lender as at the second, because a co-lender that monitors also passes signals to the shocked bank, which then cuts less. Without the register, the two co-lenders' euro cuts would be equal, 0.63 and 0.61 percent of their pre-shock lines.

The illustration also shows the remaining results at work. With the co-lender without monitoring of its own, neither the within-firm gap nor its weighted version measures the firm's loss: $\theta = -0.82$ percentage points and $s\,\theta = -0.41$, while the firm's credit falls by 2.37 percent. Under the reporting rule of Proposition 5(d), halfway to the mean settlement date the co-lender's average reported PD rises by 0.25 percentage points when it has no monitoring of its own and by 0.12 points when it monitors as much as the shocked bank did, while the gap between the shocked bank's and the co-lender's average reported PDs falls by 0.49 and 0.50 points.

The framework leaves out several features. It covers one renewal cycle: with a persistent type, the banks' beliefs at the next renewal would differ according to what each has seen, and their lines would signal those beliefs. Monitoring is exogenous: if banks chose it, an information shock would raise the co-lender's marginal value of its own monitoring, because it receives fewer signals from the shocked bank, and lower it, because it has a smaller line to protect, so the sign of the response is ambiguous. The shocked bank supports the borrower only through its credit; monitoring that sustains the borrower's creditworthiness (Carletti,

Figure 1. The Co-Lender's Response by Its Own Monitoring

This figure plots the percent change in the co-lender's credit line after an information shock that lowers the shocked bank's monitoring from $m_A^0 = 1$ to $m_A^1 = 0.25$, and its two parts in Proposition 1, the borrower-risk channel $\sigma\,\Delta x_A$ and the information channel Δa_B, each in percent of the co-lender's line before the shock, for co-lenders that differ only in their own monitoring m_B. The dotted line marks the shocked bank's monitoring before the shock. Every co-lender cuts; the information channel shrinks with the co-lender's own monitoring much faster than the borrower-risk channel does. Table 1 reports the values at $m_B = 0$ and $m_B = 1$ and the parameters, which are illustrative, not calibrated.

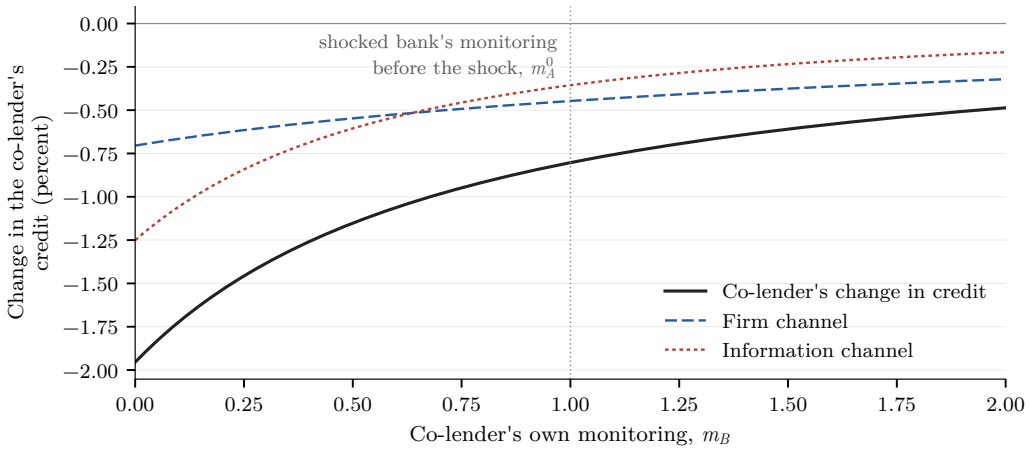


Table 1. Illustration of the Framework

This table reports the changes after an information shock that lowers the shocked bank's monitoring from $m_A^0 = 1$ to $m_A^1 = 0.25$, for a co-lender without monitoring of its own and for one that monitors as much as the shocked bank did before the shock. The firm and information channels are the two parts of the co-lender's change in Proposition 1, $\sigma\Delta x_A$ and Δa_B, in percent of its pre-shock line. Early classification is the probability P_j that a bank classifies a nonviable firm in default, and freezes its line, before settlement. Parameters: $v = 0.97$, $d = 1$, $\kappa = 0.3$, $\omega = 3$, $\varphi = 0.6$, $\psi = 0.082$ (so $\eta = v\varphi - \psi = 0.5$ and $\sigma = 0.25$), $\ell_0 = 0.26$, $r = 0.6634$, $f_A = f_B = 0.10$, $\bar{x} = 0.215$. The firm's probability of default at renewal is 5.5 percent with the co-lender without own monitoring and 5.1 percent with the other. The parameters are illustrative, not calibrated.

	Co-lender without own monitoring ($m_B = 0$)	Co-lender with $m_B = m_A^0 = 1$
Change in credit after the information shock, percent		
Shocked bank	−2.77	−1.79
Co-lender	−1.95	−0.80
Borrower-risk channel	−0.70	−0.45
Information channel	−1.25	−0.36
Within-firm gap θ	−0.82	−0.99
Firm's total credit	−2.37	−1.30
Other changes after the information shock		
ρ	0.537	0.224
Default probability (pp)	0.54	0.30
Early classification, shocked bank (pp)	−30.0	−21.0
Early classification, co-lender (pp)	−16.1	−4.7

2004) would add a second route from the shock to default. Finally, the illustration is not calibrated.

4 Proofs

Write $h_j = 1/(d+\Lambda_j^0) + 1/(d+\Lambda_j^1)$.

Proof of Lemma 1. (a) Signals arrive only from a nonviable firm, so after a signal the bank knows that the firm will draw every open line at settlement and default. Freezing avoids the loss of $x_j > 0$ and costs nothing, since the fee was paid at renewal; with $x_j = 0$ the freeze is immaterial. Without a signal the contract does not allow a freeze. The other bank's freeze changes neither the firm's default, which has fundamental causes, nor bank j's loss. (b) Bank j's own signals and the other bank's signals that reach it are independent Poisson processes with rates m_j and $\kappa m_{j'}$, so its first signal arrives at rate Λ_j. Settlement arrives independently at rate d; it comes first with probability $d/(d+\Lambda_j)$, and the freeze with probability $\Lambda_j/(d+\Lambda_j)$. (c) By a date $t < t_s$, a nonviable firm has sent bank j no signal with probability $e^{-\Lambda_j t}$, and a viable firm sends none. Settlement arrives at the same rate for both types, so the event $t_s > t$ is uninformative, and Bayes' rule gives the posterior. At settlement a nonviable firm defaults with probability one and a viable firm with probability $\ell(x)$, which gives equation (3). □

Proof of Lemma 2. By Lemma 1(b), bank j's expected profit is equation (1). Because $2\bar{x} \le \ell_0/\varphi$, the failure probability is linear on the whole domain of lines, and because $r \ge 2\psi\bar{x}$ the fee is nonnegative there (Assumption 3(b)). The expected profit is therefore $x_j(M_j + \eta x) - \omega x_j^2/2$ for every feasible pair of lines, with second derivative $-G$, and $G = \omega - 2\eta > \eta > 0$ by Assumptions 1 and 3(d). The first-order condition $M_j + \eta x + \eta x_j - \omega x_j = 0$ gives $x_j = a_j + \sigma x_{j'}$, and the best response on $[0, \bar{x}]$ is this value clipped to the interval. Since $0 < \eta < G$, $\sigma \in (0,1)$. The clipped best responses therefore form a contraction, so the equilibrium is unique. Solving the two unclipped best responses gives the equilibrium lines; $x_j > 0$ because $a_j > 0$ (Assumption 3(a)) and $\sigma > 0$, and $x_j < \bar{x}$ by Assumption 3(b). □

Proof of Lemma 3. (a) The information shock enters M_j through $-(1-v)\,d/(d+\Lambda_j)$; it lowers Λ_A by z and Λ_B by κz, and $d/(d+\Lambda^1) - d/(d+\Lambda^0) = d(\Lambda^0 - \Lambda^1)/[(d+\Lambda^0)(d+\Lambda^1)]$ gives equations (4) and (5). (b)(i) If $m_B = m_A^0$, then $\Lambda_A^0 = \Lambda_B^0$, and as $z \to 0$ both Λ^1 tend to Λ^0. (ii) $\partial \ln \rho / \partial m_B = \kappa h_A - h_B$. Since $\kappa < 1$, $\kappa(d+\Lambda_B^n) = \kappa d + \kappa m_B + \kappa^2 m_A^n < d + m_A^n + \kappa m_B = d + \Lambda_A^n$ for $n \in \{0,1\}$, so $\kappa/(d+\Lambda_A^n) < 1/(d+\Lambda_B^n)$ and $\kappa h_A < h_B$. (iii) By (ii), ρ is largest at $m_B = 0$, where $\rho < 1$ if and only if $(1-\kappa)(d^2 - \kappa m_A^0 m_A^1) > 0$; Assumption 2 gives $m_A^1 < m_A^0 \le d$. □

Proof of Proposition 1. By Lemma 2, $\Delta x_A = (\Delta a_A + \sigma \Delta a_B)/(1-\sigma^2)$ and $\Delta x_B = (\Delta a_B + \sigma \Delta a_A)/(1-\sigma^2)$, exactly, for any shock that keeps Assumption 3. With $\Delta a_B = \rho \Delta a_A$ and $\Delta a_A < 0$, both are negative because $\rho > 0$ and $\sigma \in (0,1)$, which gives the first equality in equation (6); since equilibrium lines are positive, the signs are the same in percent. The best responses $x_B = a_B + \sigma x_A$ and $x_A = a_A + \sigma x_B$ hold in both equilibria; subtracting gives the second equality and $\Delta x_A = \sigma \Delta x_B + \Delta a_A$. Both parts are negative: $\sigma \Delta x_A < 0$, and $\Delta a_B = \rho \Delta a_A < 0$. With $\kappa = 0$, $D_B = 0$, so $\Delta a_B = 0$ and $\Delta x_B = \sigma \Delta x_A < 0$. □

Proof of Proposition 2. $\Delta x_A - \Delta x_B = (1-\sigma)(\Delta a_A - \Delta a_B)/(1-\sigma^2) = (1-\rho)\Delta a_A/(1+\sigma) < 0$ by Lemma 3(b)(iii). Its ratio to Δx_A is $(1-\rho)(1-\sigma)/(1+\sigma\rho)$, which is positive and below one, since $(1-\rho)(1-\sigma) < 1 < 1+\sigma\rho$. Then $\Delta x = \Delta x_A + \Delta x_B = (\Delta x_A - \Delta x_B) + 2\Delta x_B$, and $\Delta x_B < 0$. Finally, the first-order condition can be written $x_j = (M_j + \eta x)/(\omega - \eta)$, with the same coefficient on the firm's total credit for both banks, so $\Delta x_A - \Delta x_B = (\Delta M_A - \Delta M_B)/(\omega - \eta)$: the gap keeps only the difference between the two banks' information losses. □

Proof of Remark 1. Since $\Delta a_A/(1-\sigma^2) < 0$ and both lines are positive, $\Delta^{\mathrm{CB}} < \Delta^{\mathrm{CL}}$ if and only if $(1+\sigma\rho)(a_B + \sigma a_A) > (\rho + \sigma)(a_A + \sigma a_B)$, that is, $(1-\sigma^2)a_B > (1-\sigma^2)\rho\, a_A$, or $\rho < a_B/a_A$; with $\Delta a_B = \rho \Delta a_A$ and $\Delta a_A < 0$, this is $\Delta a_A/a_A < \Delta a_B/a_B$. Since $\Delta^{\mathrm{CL}} < 0$ (Proposition 1), $\Delta^{\mathrm{CB}} < \Delta^{\mathrm{CL}}$ puts $\theta = \Delta^{\mathrm{CB}} - \Delta^{\mathrm{CL}}$ between Δ^{CB} and zero. By equation (7), $\Delta x/x = s\,\theta + \Delta^{\mathrm{CL}}$, which is below $s\,\theta$ because $\Delta^{\mathrm{CL}} < 0$. Finally $x = (a_A + a_B)/(1-\sigma)$, so $\Delta x/x = (\Delta a_A + \Delta a_B)/(a_A + a_B) = (\Delta M_A + \Delta M_B)/(M_A + M_B)$. □

Proof of Proposition 3. By Proposition 1 and Lemma 3, the co-lender's cut is $(1-v)(D_B + \sigma D_A)/[G(1-\sigma^2)] > 0$. Since $\partial D_B/\partial m_B = -D_B h_B < 0$ and $\partial D_A/\partial m_B = -\kappa D_A h_A < 0$, it falls with m_B. The pre-shock line $x_B = (a_B + \sigma a_A)/(1-\sigma^2)$ rises with m_B, since both Λ_B^0 and Λ_A^0 rise with it and $\sigma > 0$; so the cut in percent falls too. The borrower-risk channel's share of the cut is $\sigma \Delta x_A/\Delta x_B = \sigma(1+\sigma\rho)/(\rho+\sigma)$, whose derivative in ρ is $\sigma(\sigma^2-1)/(\rho+\sigma)^2 < 0$; since ρ falls with m_B (Lemma 3), the information channel's share falls with m_B. With $\kappa = 0$, $D_B = 0$ and $\Lambda_A = m_A$, so $\Delta x_B = \sigma \Delta a_A/(1-\sigma^2) < 0$, independent of m_B. □

Proof of Proposition 4. (a) $\Delta x_B = -(1-v)(D_B + \sigma D_A)/[G(1-\sigma^2)]$ does not involve f_A or f_B, and neither do its two parts. Lowering f_A and raising f_B by the same amount raises a_A and lowers a_B by the same amount, so x_A rises and x_B falls by that amount divided by $1+\sigma$, and x does not change; so s rises as x_B falls, and $|\Delta x_B|/x_B$ rises, through each channel. If only f_A falls, x_B rises, since $\partial x_B/\partial a_A = \sigma/(1-\sigma^2) > 0$, and the cut in percent falls. (b) With

m_A^1 fixed, m_A^0 enters only the pre-shock terms, with $\partial a_A/\partial m_A^0 = (1-v)d/[G(d+\Lambda_A^0)^2] > 0$ and $\partial a_B/\partial m_A^0 = (1-v)d\kappa/[G(d+\Lambda_B^0)^2] > 0$, so both pre-shock lines rise, since $\sigma > 0$. Since $\partial D_A/\partial m_A^0 = d/(d+\Lambda_A^0)^2 > 0$ and $\partial D_B/\partial m_A^0 = \kappa d/(d+\Lambda_B^0)^2 > 0$, both banks' cuts in euros grow, and so do the borrower-risk channel $\sigma\Delta x_A$ and the information channel $\Delta a_B = -(1-v)D_B/G$. In percent, each bank's response is $x_j^1/x_j^0 - 1$, where the post-shock line depends only on m_A^1 and the pre-shock line rises with m_A^0; so both percent cuts grow. The share rises if and only if $x_B\,\partial x_A/\partial m_A^0 > x_A\,\partial x_B/\partial m_A^0$, which reduces to $a_B\,\partial a_A/\partial m_A^0 > a_A\,\partial a_B/\partial m_A^0$, the stated condition. The rise in the default probability, $v\varphi|\Delta x| = v\varphi(1-v)(D_A+D_B)/[G(1-\sigma)]$, increases with m_A^0. □

Proof of Proposition 5. (a) $\Delta x = (\Delta a_A + \Delta a_B)/(1-\sigma) = (1+\rho)\Delta a_A/(1-\sigma) < 0$. At renewal both banks hold the common belief, and by Lemma 1(c) at $t=0$ the probability of default is $1-v+v\,\ell(x)$, which changes by $-v\varphi\Delta x$. (b) $P_j = 1 - d/(d+\Lambda_j)$, so $\Delta P_j = -D_j$ and $\Delta P_A - \Delta P_B = -(1-\rho)D_A < 0$. If $m_B = m_A^0$, after the shock $\Lambda_B^1 - \Lambda_A^1 = (1-\kappa)z > 0$, so $n_A(t) > n_B(t)$ at every $t > 0$. $PD_j(t)$ increases in n_j because $v\ell < v$, so the shocked bank's probability of default conditional on no signal is higher, and on a borrower neither bank has classified, which includes every viable firm, it is the higher one. On nonviable firms the average is $(1-e^{-\Lambda_j t}) + e^{-\Lambda_j t}PD_j(t) = 1 - v(1-\ell)/[(1-v)+v\,e^{\Lambda_j t}]$, which increases in Λ_j. Over all borrowers before settlement, the average is $(1-v)(1-e^{-\Lambda_j t}) + n_j + v\ell = 1 - v + v\ell$ at both banks, as the law of iterated expectations requires. (c) Conditional on $t_s > t$, a bank's expected squared forecast error is the expected conditional variance of the default indicator given its information. Classified borrowers default for sure; unclassified borrowers have mass $n_j + v$ and default probability PD_j. Over all borrowers the error is therefore $(n_j+v)PD_j(1-PD_j) = v(1-\ell)(n_j+v\ell)/(n_j+v)$, and over unclassified borrowers it is $PD_j(1-PD_j)$. The first increases in n_j and, as long as $PD_j < 1/2$, in ℓ; the second increases in PD_j as long as $PD_j < 1/2$, and PD_j increases in both n_j and ℓ. The condition $PD_j < 1/2$ holds because $PD_j(t) \le PD_j(0) = 1 - v + v\ell \le 1 - v + v\ell_0 < 1/2$ (Assumption 3(c)). The shock raises ℓ by part (a) and, at every $t > 0$, both n_j; at $t = 0$, $n_j = 1 - v$ at both banks and only ℓ moves. If $m_B = m_A^0$, the errors are the same functions of n_j with the same ℓ; they are equal before the shock, and after it $n_A > n_B$ at every $t > 0$. (d) Under the rule, bank j reports the renewal probability of default $PD_0 = 1 - v + v\,\ell(x)$, common to both banks, on every borrower it has not classified, and one on the nonviable borrowers it has classified, whose mass is $(1-v)(1-e^{-\Lambda_j t})$ conditional on $t_s > t$. Its average report is therefore $PD_0 + (1-PD_0)(1-v)(1-e^{-\Lambda_j t})$, and the shocked bank's minus the co-lender's is $(1-PD_0)(1-v)(e^{-\Lambda_B t} - e^{-\Lambda_A t})$. If $m_B = m_A^0$, this is zero before the shock and negative after it at every $t > 0$, because $\Lambda_A^1 < \Lambda_B^1$. □

References

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